Recursive stack to zero offset along local slope

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Abstract

Velocity-independent seismic data processing requires information about the local slope in the data. From estimates of local time and space derivatives of the data, a total least squares algorithm gives an estimate of the local slope at each data point. Total least squares minimizes the orthogonal distance from the data points (the local time and space derivatives) to the fitted straight line defining the local slope. This gives a more consistent estimate of local slope than standard least squares, since it takes into account uncertainty in both the temporal and spatial derivatives.

The new slope estimation algorithm is applied to stacking along local slope. Starting at the largest offset, the estimated signal is accumulated until the smallest offset without using velocity information. Extrapolation to zero offset is done using a hyperbolic traveltime function where slope information replaces the knowledge of the NMO velocity. The new data processing method requires no velocity analysis or mute and there is no stretch effect. All major reflections and diffractions which are present at zero offset will be reproduced in the output zero-offset section. It therefore requires previous multiple removal if multiple reflections are undesired in the output.

In the case that the NMO velocities for the primary reflections are known, this can be used to produce a map of local slope. Stacking along these slopes produces a better estimate of the zero-offset primary reflections, again without stretch effect.

Synthetic and field seismic data examples demonstrate the effectiveness of the method. Comparison with standard seismic data processing, velocity analysis, mute, NMO correction and stack, shows that the new method is superior in complex data sets.

Introduction

In conventional seismic data processing (Yilmaz, 1987) normal-moveout (NMO) velocities are estimated from common-midpoint (CMP) data gathers and used in NMO corrections before the data are stacked to produce an estimate of the zero-offset section. The choice of the NMO velocities for the primary reflections (velocity picking) is tedious and requires interpretation.

Velocity-independent seismic time processing uses local slope information (Ottolini, 1983, Fomel, 2007) instead of NMO velocities. The seismic data are analysed according to local slope which must be estimated. This can be done by plane-wave destruction (Fomel, 2002) or by semblance-based methods (Marfurt, 2006). A survey is given by Schleicher et al (2009).

We first estimate the local time and space derivatives of the data using a method given by Melo et al (2009) with an interpolation algorithm described by Shepard (1968). These local derivatives will be used to estimate the local slope $p = dt/dx$ at each data point. As both the time derivatives and the space derivatives have errors, total least squares is used to estimate the local slope as described in Porsani et al (2013). In total least squares we minimize the orthogonal distance from the data points to the fitted straight line. Markovsky and Van Huffel (2007) discuss many technical issues of total least squares and there are numerous references.

The new slope estimation method can be used in a number of applications for velocity-independent seismic time processing, including stacking along local slope (Fomel, 2007) pre-stack time migration (Cooke et al, 2009), stereotomography (Lambaré, 2006) or dip-adaptive filtering (Porsani et al, 2013).

Here we shall demonstrate it on stacking along local slope. This will enhance all strong coherent events in the data, like reflections and diffractions. If multiple reflections are undesired, they should be attenuated before the stacking process is applied. This can be done with a surface-related-multiple elimination algorithm (Verschuur et al, 1992) and/or with parabolic Radon transform filtering (Abbad et al, 2011). The result of the stacking process is an estimate of the signal at the least recorded offset. This signal is extrapolated to zero offset using an equation similar to the one used by Ottolini (1983) or Fomel (2007).

We propose to compute a local slope field which corresponds to the estimated NMO velocities of the primary reflections. Stacking along these slopes yield a stack section with the correct NMO velocities and reduced presence of multiple reflections. And the slope stack has very small stretch effects.

The different processing schemes are illustrated with synthetic and field data examples.

Estimation of local slope

We have data $d(t_i, x_j)$ for $t_i, i = 1, \ldots, N_t$ and $x_j, j = 1, \ldots, N_x$. First we estimate the local time and space derivative of the data, Dt_k and Dx_k, using the method described by Melo et al (2009).

We expand the data $d(t,x)$ in a Taylor series

$$d(t+\Delta t, x+\Delta x) = d(t,x) + \frac{\partial d}{\partial t}\Delta t + \frac{\partial d}{\partial x}\Delta x \tag{1}$$

For a plane wave $d(t+\Delta t, x+\Delta x) = d(t,x)$ so that

$$\frac{\partial d}{\partial t}\Delta t + \frac{\partial d}{\partial x}\Delta x = 0 \tag{2}$$

With $p = dt/dx$ and using the derivatives

$$pDt_k + Dx_k = 0 \tag{3}$$

where the index k covers a suitable domain around the (t,x) value we want to estimate p. Using standard least squares we minimize

$$\phi_t = ||e_t||^2 = \sum_k |pDt_k + Dx_k|^2 = ap^2 + 2cp + b \tag{4}$$

with $a = \sum_k |Dt_k|^2$, $b = \sum_k |Dt_x|^2$, $c = \sum_k Dt_k Dx_k$.

The result is

$$\hat{p} = -\frac{c}{a} \tag{5}$$

Alternatively we may estimate $q = dx/dt$ by minimizing, see $Fig.1$,

$$\phi_x = ||e_x||^2 = \sum_k |qDx_k + Dt_k|^2 = bq^2 + 2cq + a \tag{6}$$

The result is

$$\hat{q} = -\frac{c}{b} \tag{7}$$

We note that

$$\hat{p}\hat{q} = \frac{c^2}{ab} \tag{8}$$

Ideally this product should be one, as $(dt/dx)(dx/dt) = 1$.

In total least squares one minimizes the sum of the squared distances from the data points to the line $dt = pdx$. As seen from Fig. 1 this corresponds to minimizing

$$\phi_p = ||e||^2 = ||e_t||^2 cos^2\theta = \frac{||e_t||^2}{1+p^2} \tag{9}$$

since $p = tan\,\theta$. Using equation 4, $\partial\phi/\partial p = 0$ gives

$$p = -\frac{1}{2c}\left[(b-a) + \sqrt{(b-a)^2 + 4c^2}\right] \tag{10}$$

Alternatively, we may minimize, see Figure 1,

$$\phi_q = ||e||^2 = ||e_x||^2 sin^2\theta = \frac{||e_x||^2}{1+q^2} \tag{11}$$

since $q = cot\,\theta$. Using equation 6, $\partial\phi_q/\partial q = 0$ gives

$$q = -\frac{1}{2c}\left[(a-b) + \sqrt{(a-b)^2 + 4c^2}\right] \tag{12}$$

We note that $pq = 1$, as it should be.

Van Huffel and Vandervalle (1991) define total least squares via SVD, analysing the problem

$$\mathbf{C}\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} Dt_k & Dx_k \end{bmatrix}\begin{bmatrix} p \\ 1 \end{bmatrix} \approx 0 \tag{13}$$

We solve the eigenvalue problem (Porsani et al, 2013)

$$\mathbf{C}^T\mathbf{C}\begin{bmatrix} p \\ 1 \end{bmatrix} = \begin{bmatrix} a & c \\ c & b \end{bmatrix}\begin{bmatrix} p \\ 1 \end{bmatrix} = \sigma^2\begin{bmatrix} p \\ 1 \end{bmatrix} \tag{14}$$

for the smallest eigenvalue σ^2. The solution is

$$\sigma^2_{\pm} = \frac{1}{2}\left[(a+b) \pm \sqrt{(a-b)^2 + 4c^2}\right] \tag{15}$$

Choosing the smallest value we obtain

$$p' = p'_- = \frac{-c}{a - \sigma^2_-} = \frac{-2c}{a - b + \sqrt{(a-b)^2 + 4c^2}} \tag{16}$$

and

$$q' = -p'_+ = \frac{-c}{-a + \sigma^2_+} = \frac{-2c}{b - a + \sqrt{(b-a)^2 + 4c^2}} \tag{17}$$

We note that $p'q' = 1$ and that $p' = p$ and $q' = q$. That is, there are unique solutions for p and q, given by equations 10 and 12 or by equation 16 and 17.

Stacking along local slope

We want to compute an estimate of the seismic trace at zero offset, $x = 0$, including all dominant waves, both reflections and diffractions. We start at the far offset and build up a stack by adding contributions from each trace using the local sloope estimate. At (t_i, x_j) we want to extrapolate the accumulated stacked signal to x_{j-1} and add it to the recorded data there. Using local slope at (t_i, x_j) time is extrapolated to

$$t = t_i + p(x_j - x_{j-1}) = t_{i'} - \beta(t_{i'} - t_{i'-1})$$

where $0 \leq \beta < 1$. The accumulated stacked signal $\hat{d}(t_i, x_j)$ is linearly distributed between $t_{i'}$ and $t_{i'-1}$ and added to the recorded signal, see Fig. 2. The accumulated signal at x_{j-1} is

$$\begin{aligned} \hat{d}(t_{i'}, x_{j-1}) &= d(t_{i'}, x_{j-1}) + (1-\beta)\hat{d}(t_i, x_j) \\ \hat{d}(t_{i'-1}, x_{j-1}) &= d(t_{i'-1}, x_{j-1}) + \beta\hat{d}(t_i, x_j) \end{aligned} \tag{18}$$

The procedure is started at the far offset with $d(t_i, x_{Nx})$.

In a final step we want to extrapolate this signal to zero offset, $x = 0$. We use the hyperbolic traveltime function (Yilmaz, 1987)

$$T(x) = \sqrt{T(0)^2 + \frac{x^2}{v^2_{NMO}}} \tag{19}$$

so that

$$p = \frac{dT}{dx} = \frac{x}{T(x)v^2_{NMO}} \tag{20}$$

To extrapolate to zero-offset we obtain (Ottolini, 1983; Fomel, 2007)

$$T(0) = \sqrt{T(x)^2 - pxT(x)} \tag{21}$$

Stacking along the NMO velocity slope

We shall use the NMO velocity derived in a classical velocity analysis (Yilmaz, 1987), $v_{NMO}(T(0))$, to compute the local slope. For each (t,x) value we solve the equation

$$t^2 = T(x)^2 = T(0)^2 + \frac{x^2}{v_{NMO}^2(T(0))} \quad (22)$$

for $T(0)$ and $v_{NMO}(T(0))$. The result is $v_{NMO}(t,x)$ and the local slope

$$p(t,x) = \frac{x}{t v_{NMO}^2(t,x)} \quad (23)$$

Stacking along this slope-field will enhance reflections with the chosen NMO velocities.

In practice, we do not solve equation 22, but assume that the slope is slowly varying and compute it along the moveout curves (eq. 22) in steps of $\Delta T(0)$. At each offset value x, the nearest time grid point $i\Delta t$ less than $T(x)$ is given the slope p in equation 23 with the NMO velocity corresponding to $T(0)$.

Numerical results

In the numerical examples we produce three types of stack sections: NMO stack, the standard stack using primary NMO velocities for NMO corrections and summation; NMO slope stack, stack along local slope derived from NMO velocities; and automatic slope stack, stack along local slope estimated by total least squares. In the automatic slope stack procedure we have used 8 points (a 3 × 3 grid minus the center point) for the estimation of the local derivatives in the time and offset directions. For the estimation of the local slope we also used a 3 × 3 grid around the central point. A larger local grid will result in a smoother function for the local slope.

In the automatic stacking process we do not want to include waves with negative slope. We constrain the local slope to the range

$$\frac{x}{t v_{NMO,max}^2} \leq p(t,x) \leq \frac{x}{t v_{NMO,min}^2} \quad (24)$$

(In the numerical examples we used $v_{NMO,min} = 1400 m/s$ and $v_{NMO,max} = 8000 m/s$). When the estimated local slope is outside this range, the stacking process is restarted. That is, the local stack up to that point is not used in the update procedure. There are at most N_x traces which are included in the slope stack. Formally we should divide the result of the stack by the number of traces used. As will be shown in the first data example, this reduces the amplitudes too much. The amplitudes are only relative, and they do not represent the amplitudes on a zero-offset section.

In the first data example we analyse three idealized reflected waves with hyperbolic traveltimes with $(T(0), v_{NMO})$=$(0.6s, 1500m/s)$, $(1.4s, 2000m/s)$ and $(2s, 2500m/s)$ respectively. The wavelets are all a $30Hz$ Ricker wavelet with unit amplitude on all traces. This is an ideal set-up for NMO stack where the correct traveltime parameters were used. There are 95 traces with a regular spacing $\Delta x = 20m$, and offsets ranging from $120m$ to $2000m$. The time sample interval is $\Delta t = 4ms$.

The results are shown in Fig. 3 where Fig 3a shows the regular build-up of the accumulated automatic slope stack from the far offset to the minimum offset. In the last step from offset $x_1 = 120m$ to zero offset, the hyperbolic time extrapolation in equation 32 was used. In Fig 3b trace 1 shows the ideal zero-offset trace. Trace 2 shows the result of NMO stack using the true traveltime parameters. It is seen that the first reflection at $0.6s$ suffers from NMO stretch effects (Yilmaz, 1987), and that the two other reflection give a perfect zero-offset signal.

The result of the automatic slope stack in trace 3 has been normalized so that max amplitude is equal to one. The wavelets are very well reproduced; the one at $0.6s$ has a slight asymmetric appearance, but there is no stretch effect. Trace 4 shows the result of automatic slope stack with amplitudes divided by the number of traces. There is a considerable amplitude reduction due to the numerous application of linear extrapolation.

Trace 5 and 6 show the results from NMO slope stack where the correct NMO velocities were used to compute the local slope. There is a slight lack of symmetry in the pulse at $0.6s$, but there is no stretch. There is a reduction in amplitude as compared with NMO stack, similar to the automatic slope stack.

In the next data example we consider the Hess 2D VTI model. It consists of a buried salt body at the left part of the section and a major fault at the right part. There are no free-surface multiples in the data. There are 3552 CMP gathers with 131 traces. The minimum offset is 67 m, the trace spacing is 67 m, and the maximum offset is 8730 m. The time sample interval is 6 ms.

We first analyse CMP 900 which is shown in Fig. 4a. Fig 4b shows the build-up of the cumulative automatic slope stack. It is shown that the stacking process removes considerable amount of noise, and that many coherent event will not be mapped to zero offset. When compared with the CMP data in Fig. 4a, it is seen that these events are not present on the near-offset traces. In NMO stack these events will appear, because they are strong at medium offsets. Fig. 4c shows the estimated slopes using total least squares, and Fig 4d shows the local slope computed from NMO velocities.

The final stack section are shown in Fig. 5. Fig. 5a shows the zero-offset data, Fig. 5b shows the result of automatic slope stack without mute. Fig. 5c shows the result of NMO slope stack without mute and Fig. 5d shows the result of NMO stack with mute applied to the data. It is seen that the automatic slope stack preserves most of the events on the zero-offset section, while NMO stack shows many events which are not present there.

Conclusions

We have developed a new method for estimating local slope using local derivatives and total least squares. The method may be applied to many forms for seismic time processing, including pre-stack time migration, dip-adaptive filtering and stereotomography.

The application to stacking along local slope resulted in a method for automatic generation of a zero-offset section, including all major events, reflections and diffractions. No velocity analysis or mute is needed and there is no stretch effect. The method does not give true-amplitude sections, but relative amplitudes are preserved.

In order to enhance only primary reflections, local slope may be computed from the NMO velocities for the desired reflections. This results in a NMO slope stack with similar results as NMO stack, but without stretch effects.

The first synthetic data example shows that both slope stack processes give results without stretch effects, but only relative amplitudes. The second synthetic data example shows that the automatic slope stack results in a section very close to zero-offset data, and that the NMO slope stack gives better results than NMO stack for early times. For later times they are very similar.

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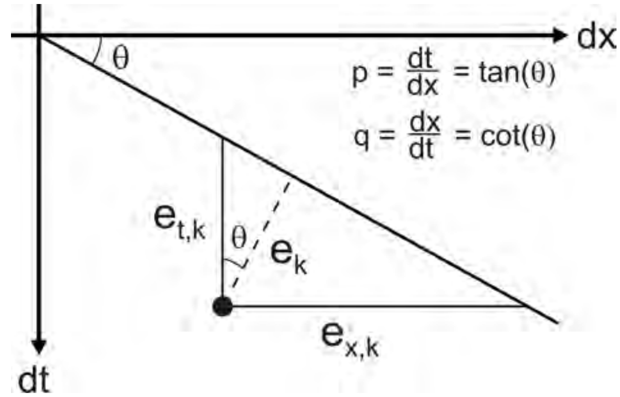

Figure 1: Geometry of total least squares estimation.

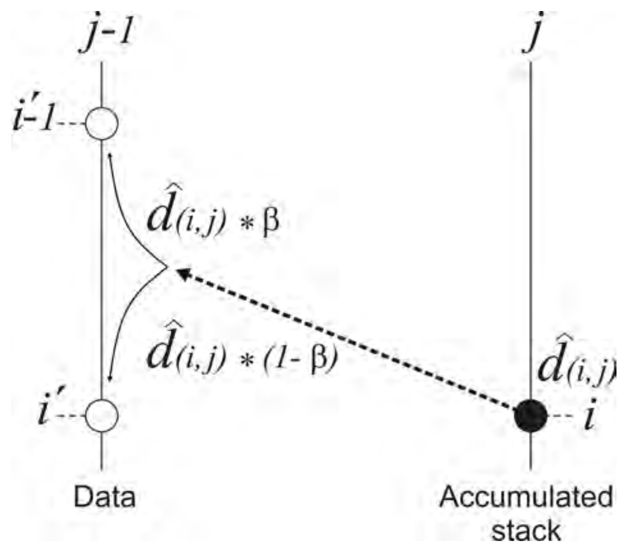

Figure 2: Update of the accumulated stack at x_{j-1}.

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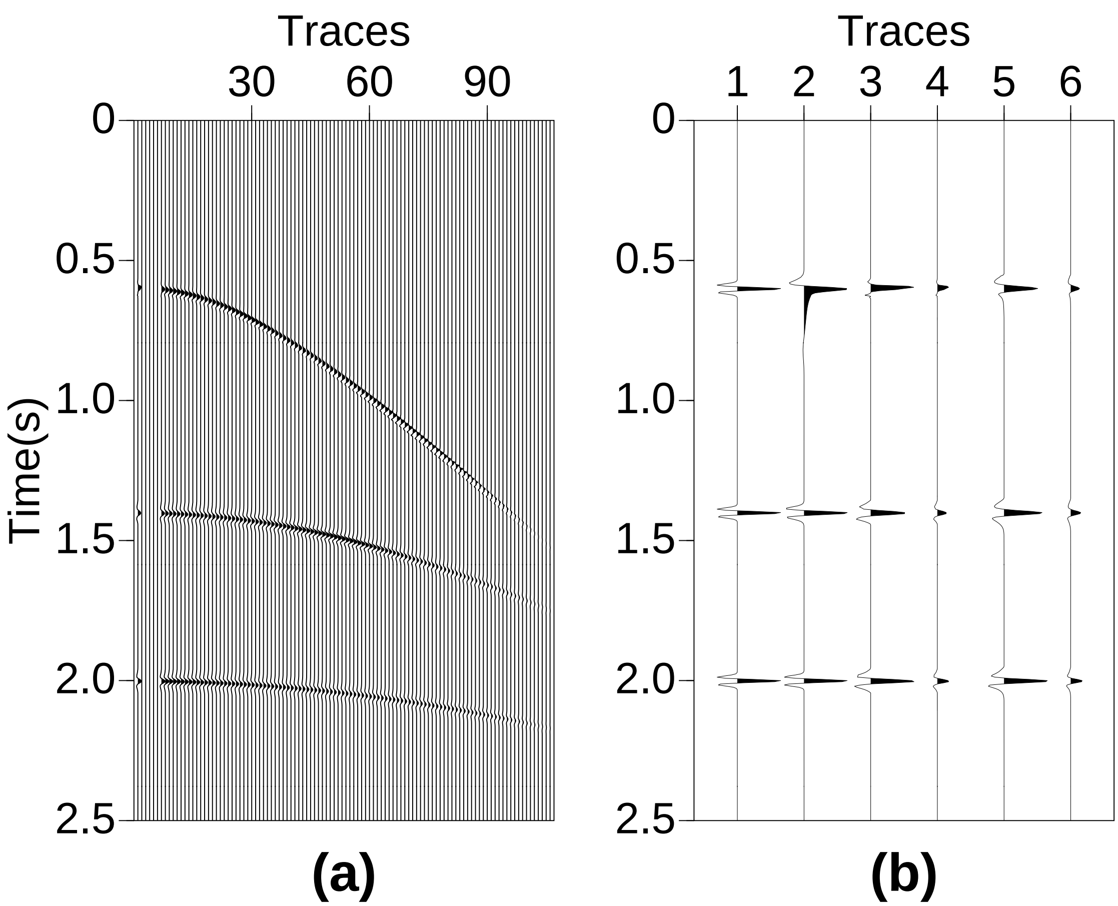

Figure 3: Ideal data example. (a) Accumulated seismic traces in the automatic slope stack. (b) Zero-offset seismic traces: Trace 1, the zero-offset trace; trace 2, the result from NMO stack; trace 3, the result from automatic slope stack, normalized amplitude; trace 4 automatic slope stack divided by the number of traces; trace 5, the result from NMO slope stack normalized amplitude; trace 6, NMO slope stack divided by the number of traces.

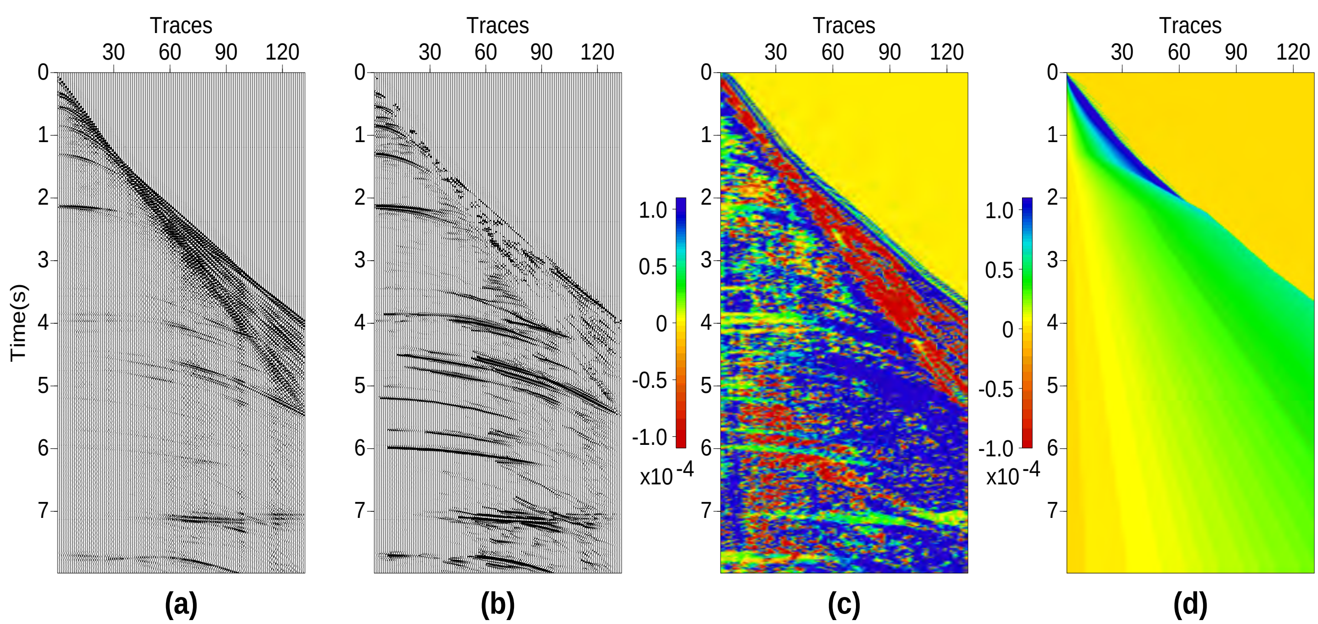

Figure 4: CMP 900 of the Hess 2D VTI data set. (a) The CMP data. (b) The cumulative build-up of the automatic slope stack. (c) Total least squares estimate of local slope. (d) Local slope computed from NMO velocities.

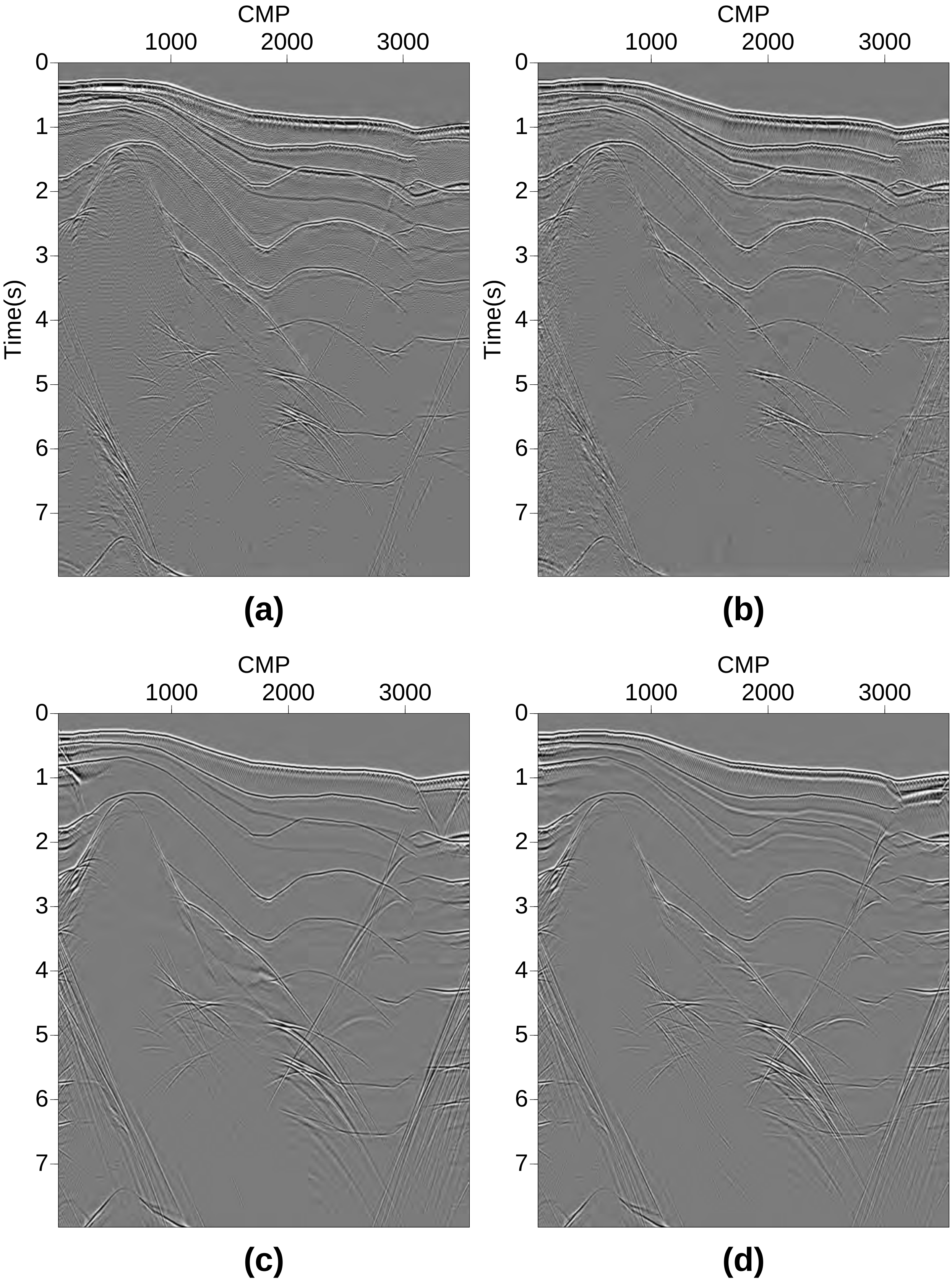

Figure 5: The Hess 2D VTI data set. (a) Zero-offset section data. (b) Result from automatic slope stack, no mute. (c) Result from NMO slope stack, no mute. (d) Result from NMO stack with mute.